

Std. 11
23-9-2016

Half Yearly Examination in **MATHEMATICS**

Time : 3 hrs.
M. Marks : 100

GENERAL INSTRUCTIONS:

- i) Attempt all the questions.
- ii) Section - A consists of 4 questions of 1 mark each.
- iii) Section - B consists of 8 questions of 2 marks each.
- iv) Section - C consists of 11 questions of 4 marks each.
- v) Section - D consists of 6 questions of 6 mark each.

SECTION – A

1. Find the multiplicative inverse of i.
2. Find the sum to infinity $8+4\sqrt{2}+4+\dots$
3. Evaluate $\frac{\cos(2\pi+\theta)\operatorname{cosec}(2\pi+\theta)\tan(\frac{\pi}{2}+\theta)}{\sec(\frac{\pi}{2}+\theta)\cos\theta\cot(\pi-\theta)}$

4. If ${}^{n-1}P_3 : {}^nP_4 = 1:9$, find n.

SECTION – B

5. If $(\frac{1+i}{1-i})^m = 1$, find the least positive integral value of m.
6. Solve for real x and represent the solution in number line.
$$\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$$
7. Using the principle of mathematical induction prove that $3^{2n} - 1$ is divisible by 8. $\forall n \in \mathbb{N}$.
8. If the sum of n terms of an A.P is $nP + \frac{1}{2}n(n-1)Q$, where P and Q are constants.
Find common difference.
9. The fourth term of G.P is square of the second term and the first term is -3.
Determine its 8th term.
10. If $\sin\theta = -\frac{4}{5}$ and θ lies in third quadrant, find the value of $\cos\frac{\theta}{2}$.
11. Prove that $\frac{\sin 7x - 2\sin 5x + \sin 3x}{\cos 7x - \cos 3x} = \tan x$.
12. Prove that $\frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = \tan 56^\circ$.

SECTION – C

13. Find the square root of complex number $1 + i$.
14. Using the principle of mathematical induction prove that
$$1.3 + 2.3^2 + 3.3^3 + \dots + n.3^n = \frac{(2n-1)3^{n+1} + 3}{4}, \quad \forall n \in \mathbb{N}.$$
15. A bag contains 6 white and 5 red marbles. Find the number of ways in which four marbles can be drawn if they can be of
(i) any color (ii) two white and two red (iii) all are of same color.
16. Prove that : $\cos 2x \cos \frac{x}{2} - \cos 3x \cos \frac{9x}{2} = \sin 5x \sin \frac{5x}{2}.$
17. Solve : $2\sin^2 x + 3\cos x = 0.$
18. If $\tan A = \frac{1 - \cos B}{\sin B}$, then prove that $\tan 2A = \tan B.$
19. Find the sum of n terms of the series: $3 + 5 + 9 + 15 + 23 + \dots$.
20. Find the real value of θ for which the expression $\frac{1 + i\cos\theta}{1 - 2i\cos\theta}$ is purely real.
21. A window in the form of a rectangle is surmounted by a semi circular opening. If the length of the window is twice of its breadth and the total perimeter is at least 368cm. What is the minimum length of the window? Why windows are important in our house?
22. The ratio of the A.M and G.M of two positive numbers a and b is $m : n$, show that
$$a : b = m + \sqrt{m^2 - n^2} : m - \sqrt{m^2 - n^2}.$$
23. Using principle of mathematical induction prove that
$$1^2 + 2^2 + 3^2 + \dots + n^2 > \frac{n^3}{3}, \quad \forall n \in \mathbb{N}.$$

SECTION – D

24. Find the polar form of the complex number $Z = \frac{1+7i}{(2-i)^2}$.
25. If S be the sum, P be the product and R be the sum of reciprocals of n terms in G.P. Prove that $P^2 R^n = S^n$.
26. Find the number of different 8-letter words that can be formed from the letters of the word INTERMEDIATE. In how many of these arrangements
- do all the words start with R
 - do All vowels occur together
 - do all vowels do not occur together.
 - do the word begin with N and ends with M.
27. If $\tan(\pi \cos \theta) = \cot(\pi \sin \theta)$. Prove that $\cos\left(\theta - \frac{\pi}{4}\right) = \pm \frac{1}{2\sqrt{2}}$.
28. Solve the following system of linear in equations graphically.
 $x + 2y \leq 3, 3x + 4y \geq 12, y \geq 1, x, y \geq 0$.
29. Show that $\frac{1 \times 2^2 + 2 \times 3^2 + \dots + n \times (n+1)^2}{1^2 \times 2 + 2^2 \times 3 + \dots + n^2 \times (n+1)} = \frac{3n+5}{3n+1}$.

-X-X-X-X-X-X-X-